University of Texas Bulletin

No. 2706: February 8, 1927

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XI, No. 2

PUBLISHED BY
THE UNIVERSITY OF TEXAS
AUSTIN

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UNIVERSITY OF TEXAS PRESS, AUSTIN

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PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH, AND ENTERED AS SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XI, No. 2

Edited by

C. D. RICE
Associate Professor of Applied Mathematics

and

MARY E. DECHERD
Adjunct Professor of Pure Mathematics

This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

CONTENTS

FOREWORD

The high school as it has been developed in our educational system is a unit that has been set off as an independent element and is not related as it should be to the work of the grades below or to the college that follows. Many new subjects are introduced in the high school that should be more closely related to the grades below. The break between high school and the grades in methods and subject matter is in many cases as striking as that between high school and college.

Some of the difficulties found in mathematics by students in the high school may be noticed:

(1) The use of letters to represent numbers.
(2) The symbols of operation.
(3) The negative number.
(4) Logical proofs about geometrical magnitudes and properties about which the student has not had sufficient preparation.

The lack of correlation between high school and grades is felt especially by the pupil in making the transition from arithmetic of definite numbers to the general statements of algebra. In a great majority of the high schools of our State this transition is made without any preparation whatever in the use of letters to represent numbers in the last year of arithmetic. This transition should be made slowly and intelligently. The student should become accustomed to the use of letters and be able to think mathematically in terms of such symbols. This work in arithmetic should not be dignified by the word algebra. A better name would be literal arithmetic. At this place may be given simple equations with sets of selected problems in which negative numbers are not involved. It is easily seen that we may use letters to great advantage in the subject of percentage, interest, proportion, mensuration, and many of the simpler business problems.

A preliminary training in the use of letters in the last year of the grades will prepare the student for the beginning of formal algebra in the first year of the high school. The negative number which is so difficult to the beginner may then be given and formal algebra introduced. By thus leading the student into one difficulty at a time he is able to progress more intelligently than when the double abstraction of letters and negative numbers are given at the same time. This pedagogical mistake is made so far as we know in no other country except in our American schools.

Another pedagogical error made in our schools is to define for the first time certain geometrical magnitudes in the second or third year of the high school and then begin in the same lesson to give logical proofs of properties and relations of those quantities. In educational systems of other countries two to three years preliminary training is given before logical or formal proofs are attempted. This is generally done by definitions, drawing, constructions, measurements and considerable training in the computation of areas and volumes. There are many constructions, measurements, and drawings that should be made to familiarize the student with the quantities about which he is to make later logical deductions.

The various attempts to introduce intuitional or objective geometry in the grades below and lead in this way through constructions and measurements to logical deductions has, in general, not become a success in our school systems. On account of this omission we have been forced to delay in our high schools the study of deductive geometry until the student has acquired a sufficient mental maturity to make any progress at all with the limited amount of material given in high-school texts. By placing geometry so late in the curriculum, algebra has been forced into the first two years of the high school and often completed when the student is least mature.

When the student is not mature enough to understand the logical processes necessary in algebra, it becomes to him a piece of mechanism that will turn out results, but to him there is no real training. Work under such conditions is a loss in time and energy for both pupil and teacher. For this reason there is a growing feeling that the last year in algebra should come later in the curriculum than now shown in the majority of our high schools.

Another error we make pedagogically is that we teach each branch of mathematics too often without any relation it may have to other subjects in mathematics. While one is studied the others are forgotten. The practice of teaching in the so-called "closed compartments" is in great contrast with the correlated methods of other countries in which the relations of the various branches are shown at each step and the student is ready to use, intelligently, at any time what he has already learned in another branch.

In the courses of study noticed in this bulletin it will be shown how the schools are meeting and overcoming the various difficulties in curriculum. We shall be glad to receive for our next bulletin any new or ideal course of study or any suggestion that will help in the revision of high-school curricula.

REPORT OF THE NATIONAL COMMITTEE

"The National Committee on Mathematical Requirements was organized in the late summer of 1916, under the auspices of the Mathematical Association of America, for the purpose of giving national expression to the movement for reform in the teaching of mathematics, which had gained considerable headway in various parts of the country, but which lacked the power that coördination and united effort alone could give. The results of the committee's work and deliberations are presented in a report published by the Bureau of Education, Washington, D. C."

Those wishing the report may obtain it by writing to the Bureau, asking for Bulletin 1921, No. 32, and sending 10 cents for each copy.

This report is full of information and good suggestions for those wishing to take up the study of reorganization of the high-school curriculum. To meet the requirements that may be met in arranging and organizing the material in a high-school course we quote the following from the report:

> **The Junior High School.**—In view of the fact that under this form of school organization, pupils may be expected to remain in school until the end of the Junior High School period instead of leaving in large numbers at the end of the eighth school year, the mathematics of the three years of the Junior High School should be planned as a unit, and should include the material recommended. There remains the question as to the order in which the various topics should be presented and the amount of time to be devoted to each. The committee has already stated its reasons for not attempting to answer this question. The following plans for the distribution of time are, however, suggested in the hope that they may be helpful, but no one of them is recommended as superior to the others, and only the large divisions of material are mentioned.

PLAN A

First Year.—Applications of arithmetic, particularly in such lines as relate to the home, to thrift, and to the various school subjects; intuitive geometry.

Second Year.—Algebra; applied arithmetic, particularly in such lines that relate to the commercial, industrial and social needs.

Third Year.—Algebra; trigonometry; demonstrative geometry.

By this plan demonstrative geometry is introduced in the third year, and arithmetic is practically completed in the second year.

PLAN B

First Year.—Applied arithmetic (as in plan A); intuitive geometry.

Second Year.—Algebra; intuitive geometry; trigonometry.

Third Year.—Applied arithmetic; algebra; trigonometry; demonstrative geometry.

By this plan trigonometry is taken up in two years, and the arithmetic is transferred from the second to the third year.

PLAN C

First Year.—Applied arithmetic (as in plan A); intuitive geometry; algebra.

Second Year.—Algebra; intuitive geometry.

Third Year.—Trigonometry; demonstrative geometry; applied arithmetic.

By this plan algebra is confined chiefly to the first two years.

PLAN D

First Year.—Applied arithmetic (as in plan A); intuitive geometry.

Second Year.—Intuitive geometry; algebra.

Third Year.—Algebra; trigonometry; applied arithmetic.

By this plan demonstrative geometry is omitted entirely.

PLAN E

First Year.—Intuitive geometry, simple formulas, elementary principles of statistics, arithmetic (as in plan A).

Second Year.—Intuitive geometry, algebra, arithmetic.

Third Year.—Geometry, numerical trigonometry, arithmetic.

Senior High School.—"In the majority of high schools at the present time the topics suggested can probably be given most advantageously as separate units of a three-year program. However, the National Committee is of the opinion that methods of organization are being experimentally perfected whereby teachers will be enabled to present much of this material more effectively in combined courses unified by one or more of such central ideas, functionality and graphic representation.

"As to the arrangement of material the committee gives below four plans which may be suggestive and helpful to teachers in arranging their courses. No one of them is, however, recommended as superior to the others."

Plan A

Tenth Year.—Plane demonstrative geometry, algebra.

Eleventh Year.—Statistics, trigonometry, solid geometry.

Twelfth Year.—The calculus, other elective.

Plan B

Tenth Year.—Plane demonstrative geometry solid geometry.

Eleventh Year.—Algebra, trigonometry, statistics.

Twelfth Year.—The calculus, other elective.

Plan C

Tenth Year.—Plane demonstrative geometry, trigonometry.

Eleventh Year.—Solid geometry, algebra, statistics.

Twelfth Year.—The calculus, other elective.

Plan D

Tenth Year.—Algebra statistics, trigonometry.

Eleventh Year.—Plane and solid geometry.

Twelfth Year.—The calculus, other elective.

REQUIREMENTS IN MATHEMATICS FOR HIGH-SCHOOL GRADUATION

MISS ELIZABETH DICE

The high schools of Texas virtually require two years of algebra and one year of plane geometry for graduation. For the slow or immature child, the result is two or three years of mechanical shifting of one set of symbols into another set of symbols and one or two years of rebellious memory work. For the bright or more mature child, the result is three years of unrelated mathematics.

In algebra, if the students are slow or immature, the teachers untrained, or the classes large, competitive drills, interesting equations, and thoughtful questions are discouraged. Thought problems, which should introduce algebra because they show the need for equations and mechanical manipulations, are not only a small part of the adopted textbook but also are poorly arranged, awkwardly worded, and out of date.

In plane geometry, the assignment of formal proofs of the congruency of certain angles and triangles before the students are taught to draw triangles or read angles or point out corresponding sides encourages unintelligent memory work, develops a dislike for logical reasoning, and discourages, electing fourth-year mathematics. The boys and girls who, despite the lack of a knowledge of intuitive geometry, understand the formal proofs of the textbook enjoy the rare opportunities of discussing original proofs and the different cases of these and other problems.

The majority of the teachers of mathematics in the secondary schools of Texas have had no training in the teaching of mathematics, have not gone beyond freshman mathematics in college, and have chosen the subject because the papers are easy to correct, or because mathematics combines nicely with administrative work, or because of a vacancy in this department. These teachers do not know what to stress as a foundation for higher mathematics, they can neither answer the questions nor properly direct the energy and interest of prospective mathematicians, and have to follow the textbook no matter how poor it may be.

Even those teachers who know mathematics and who, if given the opportunity to introduce new divisions of the subject or if given the same students two successive terms, know how to teach, cannot help students to remember principles which are rarely applied. Do you remember a poem which you did know but have not repeated for two years? Can you name the presidents in order—or out of order? Can the average college graduate, three months after receiving his degree, extract the cube root, arrange a magic square or solve without review any problem in calculus? Cube root, the magic square, and integration are used as frequently by college students as are factoring, radicals and quadratics by high-school students, and advanced applications are no less difficult for the college students than the elementary principles are for the students of the high school.

Texas high schools had the same requirements for graduation in 1900 yet they sent to college students who knew more algebra than the students of 1926. Why? (1) The classes are larger. "The *maim,* the lame, and the blind," mathematically speaking, are in the high schools now. They take a great deal of the time which was given to the bright children. (2) The students are less mature. The mental age exceeds the chronological age—on the surface. Algebra which will *stick* for two years is not a surface subject. (3) The children of 1926 have less time. While the children of 1900 interested themselves in solving problems in pure mathematics, the children of 1926 are studying applied mathematics: the radio, the mechanism of an automobile, wiring the garage, etc. In, fact, so many of the results of mathematics do they know and understand and appreciate that they should not only be forgiven for not knowing the underlying principles but also should be given teachers, books, laboratories, and classmates to make mathematics one of the most enjoyable courses in the secondary schools.

Some of the mathematical requirements for high-school graduation should be: teachers trained to teach mathematics, textbooks which deal with problems of this century, required and elective courses, and different classes or at least different assignments varying with the ability of the children. The ability of the child may be determined by a combination of the I. Q., grades from grammar school, and grades for the first few weeks in high school.

Two years of general mathematics in which mental arithmetic, the language of algebra, intuitive geometry, and fundamental notions of trigonometry are combined in a way which the less gifted or vocationally inclined children can understand and appreciate should be made a requirement for graduation. Half of this course may be given the third or fourth year thereby supplementing the mental age or the mathematical inaptitude with the chronological age. The more gifted children, mathematically speaking, especially prospective engineers, can take the essentials of the course in general mathematics in one year. The two groups can, with different assignments, use the same textbook for the first year. In the small high school the two groups, even though in the same class, can be given different assignments.

A course of related mathematics with more advanced algebra and formal geometry may complete the two-year requirement for the second group. The children whose work is satisfactory the second year in the second group may elect third, or fourth, or third and fourth year mathematics. The elective courses, since the children have been more or less balanced by the first two years, may be less closely related courses. Certainly the latter half of the fourth year should be a preparation for college mathematics.

The first group, that is the group not mathematically inclined, should, if their work for the first year is above the average, have the privilege of enrolling in the second group. Otherwise they should be required to take a second year in general mathematics. The second group should be shifted to the first group any time occasion may arise. Briefly, the men and women who plan the mathematical requirements for high-school graduation should shift the children intelligently rather than allow the boys and girls to shift one set of symbols into another set of symbols unintelligently.

HIGH SCHOOL COURSE OF STUDY

P. H. UNDERWOOD
Ball High School, Galveston, Texas

First Year. First Term. Applied Arithmetic. Intuitive Geometry

ARITHMETIC

I. Review fundamental operation of fractions and decimals. The meaning the terms: sum, remainder, difference, product, quotient, factor, multiple, least common multiple. The ability to express by words and by symbols any one of the terms, multiplier, multiplicand and product in terms of the other two.

The ability to express by words and by symbols any one of the terms, division, dividend, quotient, and remainder in terms of the other three.

In a simple division problem how is the quotient affected if the dividend is multiplied by a number? How is it affected if the divisor is multiplied by a number? How, if both dividend and divisor are increased in the same proportion?

Reduction of common fractions to decimals and of decimals to common fractions. An appreciation of the fact that a common fraction in its lowest terms can be exactly expressed as a decimal only when its denominator contains no other prime factors than twos and fives. Short cuts in multiplication and division by 25, 125, etc. Tests of divisibility by 3, 4, 6, 8, 9.

II. Percentage. Finding a per cent of a quantity. Finding a quantity if a per cent of it is given. Finding what per cent one quantity is of another. Finding the per cents equivalent to common fractions.

III. Drill in the formulas

$$I = prt$$
$$A = p(1 + rt).$$

IV. Ratio and proportion. Application to indirect measurement. Easy examples in drawing to scale.

V. Square root. Pythagoras' Theorem. $c^2=a^2+b^2$.

VI. Areas: Rectangle, square, parallelogram, triangle, trapezoid polygon with study of the formulas for perimeters and areas. Circumference and area of the circle.

$$p=2a+2b,\ A=lw,\ A=\tfrac{1}{2}bh,\ A=S^2,\ A=\tfrac{1}{2}(a+b)h$$
$$C=2\pi r,\ A=\pi r^2,\ A=\sqrt{s(s-a)(s-b)(s-c)}.$$

VII. Volumes and surfaces of the prism, pyramid, cylinder cone and sphere with the corresponding formulas:

$$V=lwh,\ V=Bh,\ V=\tfrac{1}{3}Bh,\ V=\pi r^2h,\ V=\tfrac{1}{3}\pi r^2h$$
$$V=4/3\pi r^3 or \tfrac{1}{6}\pi d^3,\ S=ph,\ S=\tfrac{1}{2}ph,\ S=4\pi r^2.$$

VIII. The ability to estimate mentally what the answer to a problem should be approximately.

INTUITIVE GEOMETRY

Measurement of lines. (Ruler and proctractor.)

Measure the length and breadth of a sheet of writing paper.

Measure the length and breadth of sloping part of your desk.

Measure in inches and in centimeters AC, CB, AB, and test by adding AC, CB.

Figure 1

Draw a line 10 cm. long and measure its length in inches. From the result find the number of inches in 1 centimeter.

Draw a line 5 inches long and measure it in centimeters. From the result find the number of centimeters in 1 inch.

Draw a straight line, mark by the eye its middle point, then measure the two parts and find your error.

Draw a straight line, divide it into three equal parts by the eye. Measure the parts and find the error.

Draw a line 3.5 inches long and find how often $\tfrac{5}{8}$ inch can be taken from it and determine the length of the part left over.

Draw a line 8 cm. long, and find how many times 1.3 cm. can be taken from it. Express the remainder in centimeters.

Draw on squared paper a rectangle. Measure and compare the length of its opposite sides.

Measurement of angles.

1. Measure the angles a, b (Fig. 2), and find their sum.
   What conclusion can you draw?

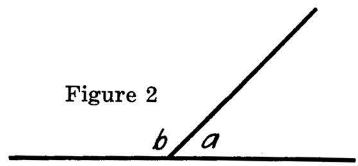


Figure 2

2. Measure the angles a, b, c, d (Fig. 3), and find their sum.
   Repeat the problem by drawing four lines starting from the same point, measuring the four angles in order and finding their sum. What inference can be drawn?

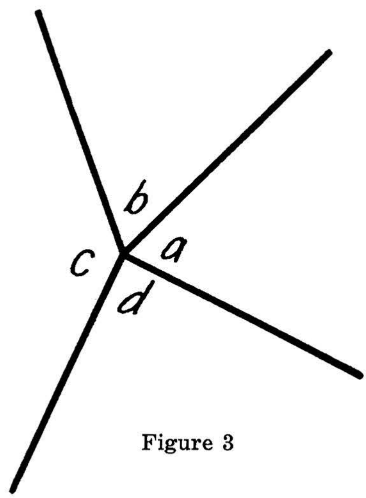


Figure 3

3. Measure and compare the angles a, b (Fig. 4); m, n. What inference can be drawn?

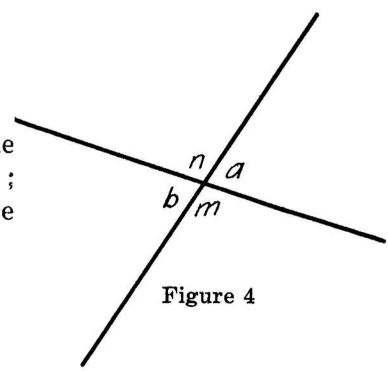


Figure 4

4. Draw angles of 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 95°, 125°.

5. Draw a triangle $A\ B\ C$ (Fig. 5) produce $A\ B$ to D. Measure the angle A and C. Find their sum. Measure also angle $C\ B\ D$. What do you find? Can you infer from this and example 1, what the sum of the angles of a triangle is?

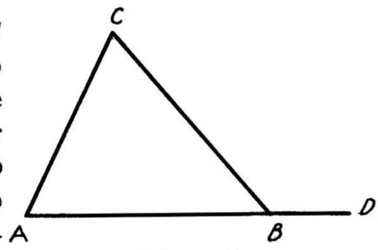


Figure 5

6. Draw two lines OM, ON making any angle and make $OM=ON$. Join MN. Measure the angles M and N. What conclusion can you draw as to the angles opposite the equal sides of a triangle?

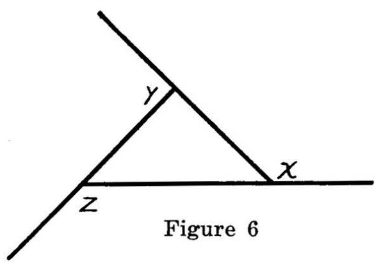


Figure 6

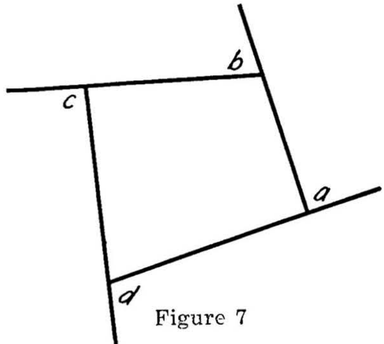


Figure 7

7. Measure the angles x, y, z (Fig. 6), made by producing the sides of the triangle in order and find their sum. Measure the angles a, b, c, d and find their sum. Draw likewise a five-sided figure (Fig. 7), produce the sides in order, measure the exterior angles and find their sum. What conclusion can be drawn? What is the sum of the angles of a polygon equal to?

8. Draw a circle (Fig. 8), and any diameter AB. On the circumference take any point C. Join CA, CB and measure angle ACB. What inference can be drawn? Draw a circle 1.5 inches radius, and in it draw a chord say 2.5 inches long. Take points M, N on one of the arcs so formed. Join MA, MB, NA, NB. Measure and compare the angles AMB, ANB. Repeat the exercise. What inference can be drawn concerning angles in a segment of a circle?

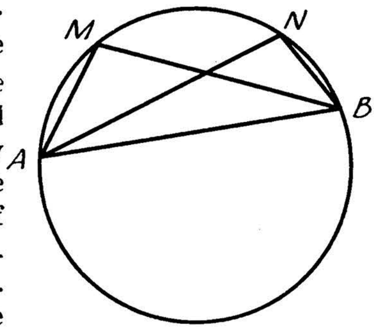


Figure 8

9. Place two lines 2½, 1½ inches so as to form 2 angles, also to form 4 angles.
10. Place 3 lines 4, 5, 6 centimeters long so as to form 2 angles, 3 angles, 4 angles, 6 angles, 9 angles, 12 angles. How many straight lines can be drawn connecting 5 points.

COMPASSES AND RULER

1. Bisect a given angle.
2. Bisect a given straight line.
3. Draw a perpendicular to a straight line from a point without it.
4. Draw a perpendicular to a straight line from a point in it.
5. Draw two lines crossing each other. Bisect two consecutive angles. Measure the angle formed by the bisectors. Can you deduce the result from a previous exercise?
6. Draw two diameters of a circle at right angles. Join their ends in order. What figure is formed.
7. Draw two diameters at right angles to each other. Bisect the four angles by radii. Join in succession the ends of the 8 radii. What figure is thus formed?

8. Draw a line $2\frac{5}{8}$ inches. Bisect it. Divide it in 4 equal parts: This gives a straight line 21/32 inch.
9. Draw an isosceles triangle. Bisect the angle formed by the equal sides and show by measurement that the bisector is perpendicular to the third side and bisects it.
10. Draw the perpendicular bisectors of the three sides of a triangle. What do you find?
11. Draw a triangle. Bisect its angles. What do you find?
12. From the vertices of a triangle draw perpendiculars to the opposite sides. What do you notice about them?
13. Draw a perpendicular to a straight line from one of its ends without producing the line. (See Fig. 9.)

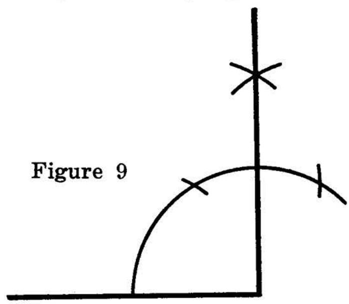


Figure 9

14. Draw a circle center O and any radius. Draw any chord AB and from O draw OM perpendicular to AB. Measure AM, MB and angles OMA, OMB. What inference may be drawn?
15. At a given point in a line make an angle equal to a given angle.
16. Draw a straight line parallel to a given line through a given point without it.
17. Draw two parallel straight lines and on one of them mark points A, B, C (See Fig. 10). Draw AM, BN, CR perpendicular to the other parallel. Measure AM, BN, CR. What conclusion can be drawn?

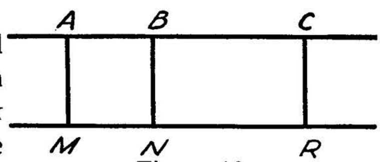


Figure 10

18. Draw a parallelogram. Measure and compare its opposite side and opposite angles.
19. Construct a triangle having given its three sides.

20. Construct a triangle given, side AB and angles A, B. Also construct a triangle given, side AB and angles B, C.
21. Given two sides and their included angle construct the triangle.
22. Two straight lines AB, CD (Fig. 11), are parallel if

angle e=angle f or
angle g=angle h

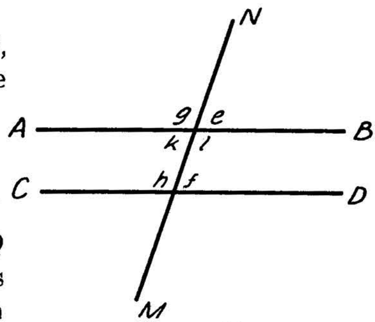


Figure 11

Show that if AB, CD are parallel and MN is any line crossing them then

angle k=angle f, angle h=angle l,
and angle l+angle f=2 right angles,
angle h+angle k=2 right angles.

23. A diagonal of a parallelogram divides the figure into two triangles equal in every respect. Prove by examples 22 and 20. Hence its opposite sides and opposite angles are equal.
24. The diagonals of a rectangle are equal. Give a reason.
25. Draw a triangle. Through the vertices draw straight lines parallel to the opposite sides. What figure is formed? How many parallelograms are formed? How many triangles equal in every respect?
26. Divide a triangle into 4 equal triangles. Hint: Join the mid points of its sides.
27. Draw a four-sided figure. Bisect its sides and join the points of bisection. What figure is formed by these 4 lines? What is the relation of each to the diagonal opposite it?
28. Divide a given straight line into 5 equal parts.
29. Construct an equilateral triangle each side being $1\frac{1}{2}$ inches.
30. Make angles of 30°, 60°, 45°.

31. Construct a square whose sides are each equal to $2\frac{3}{4}$ inches.
32. Construct a regular hexagon in a circle.
33. Construct an equilateral triangle in a circle.
34. Construct a circle about a square.
35. Describe a circle in a square.
36. Given a square, make a square half as large.

AREAS. RIGHT TRIANGLE

1. Draw a rectangle on squared paper whose length is 6 units and breadth 4 units. How many squares does it contain? Repeat the exercise for rectangles whose sides are 8 and 3 units, 9 and 5 units. What inference may be formed?
2. Show by a figure that $30\frac{1}{4}$ sq. yds.=1 sq. rd.
3. Show by a figure that $1\frac{2}{3} \times 3\frac{1}{2} = 5\frac{5}{6}$.
4. Show by a figure that $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$.
5. Change a parallelogram into a rectangle having the same base and height. From this construction deduce the rule for finding the area of a parallelogram.
6. Give a reason for the rule, the area of a triangle equals half the product of the base and height.
7. $ABCD$ is a trapezoid (Fig. 12); CE is parallel to DB triangle $BDC = \triangle BDE$ each being $\frac{1}{2}$ $BDCE$. Hence show that triangle $ADE = ABCD$. From this deduce the rule $ABCD = \frac{1}{2}(a+b)h$.

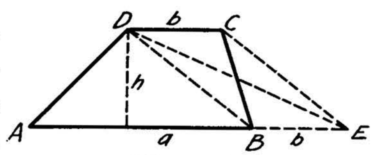


Figure 12

8. Draw on squared paper right triangles whose sides containing the right angle are

6, 8 units
5, 12 units
8, 15 units
20, 21 units

measure the hypotenuse of each and verify that the square of the hypotenuse equals the sum of the squares of the other two sides.

9. The side of a square is 10 *cm.* Find by calculation the length of a diagonal.
10. Draw an equilateral triangle *ABC,* bisect angle *C* by the straight line *CM* meeting *AB* at *M.* Find in degrees the angles of the triangle *AMC.* What is the ratio of *AM* to *AC.* If $AC=4$ in., find *AM, MC.* If one angle of a right triangle is 30° what is the relation between the side opposite this angle and the hypotenuse?
11. If *ABC* is a triangle having angle $B=90°$, angle $A=30°$ and side $BC=6$ in., what is the value of the remaining angle and the lengths of the other two sides?
12. Two equilateral triangles *ABC, ABD* stand on the same base but on opposite sides of it. If $AC=7$ in. find *CD.*
13. A room is 17 ft. long, 14 ft. wide, 10 ft. high. Find the diagonals of the walls and the diagonal of the floor.
14. The side of a square is 18 in. Find its diagonal.

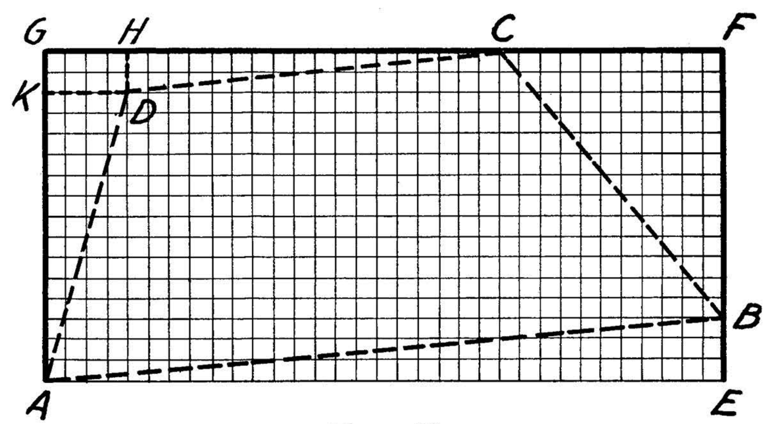


Figure 13

15. Find by calculation AB, BC, CD, DA (Fig. 13), and the area of $ABCD$, the linear unit being the side of a small square and the surface unit being the small square.

Hint: $AEFG = ABCD + AEB + BFC + CHD + DHGK + AKD$.

SYMMETRY

Its use in proving properties of figures. Show that the equilateral triangle, rectangle, square, rhombus, circle and regular polygons have one or more axes of symmetry, and that the parallelogram, square rectangle, circle and regular polygons have central symmetry.

SIMILARITY

Informal treatment of the relations of corresponding sides and of the areas of similar figures.

Construction of similar figures. *E.g.* Given $ABCDE$ (Fig. 14), and AB'. Draw $AB'C'D'E'$ similar to $ABCDE$. Show by means of a figure that

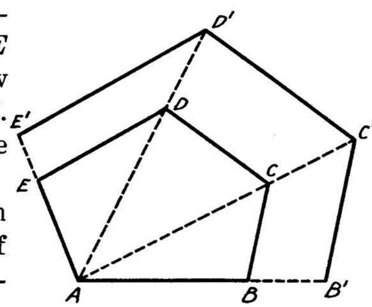


Figure 14

(i) The square on the sum of two lines equals the sum of the squares on the lines together with twice the rectangle contained by the lines.

(ii) The difference of the squares on two lines equals the rectangle contained by the sum and the difference of the two lines.

(iii) The square on the difference of two lines equals the sum of the squares on the lines diminished by twice their rectangle.

SOLIDS

What is the fewest number of plane faces a solid body can have?

Find the number of faces, the number of vertices, the number of edges of

(1) A triangular pyramid. (2) A square pyramid. (3) A cube. (4) A triangular prism. (5) A prism having six sides for base. (6) An octahedron.

Can straight lines be drawn on the curved surfaces of cylinders and cones?

What is the section of a sphere made by a plane?

What is the section of a cylinder or a cone made by a plane perpendicular to the axis?

A room is 10 ft.x10 ft. and 10 ft. high. A wasp travels from a corner of the floor to the opposite corner of the ceiling taking the shortest route. Describe its path. Can you find its length?

The edge of a cube is 2 inches. Find the diagonal of a face and the diagonal of the cube.

Show how a triangular prism may be divided into three pyramids of equal volume.

How is the volume of a solid of uniform cross-section measured?

Construct a regular hexagon whose sides will be on the six faces of a cube.

ALGEBRA

First Year—Second Term

1. Positive and negative numbers, their meaning and use, their graphic representation, the fundamental operations applied to them.
2. Linear equations in one unknown. Problems leading to linear equations which do not involve fractions.
3. The meaning, use, evolution and transformation of simple formulas involving ideas with which the pupil is familiar.

   E.g. $d=rt$. $A=\frac{1}{2}ba$. $V=lwh$. $A=\frac{1}{2}(a+b)h$.

4. Graphs of statistics.

5. Linear equations in two unknowns with verification of results.

$$E.g.\quad \tfrac{2}{3}x+\tfrac{1}{4}y=5$$
$$\tfrac{1}{2}x+\tfrac{1}{3}y=1\tfrac{1}{3}$$

6. Problems.

SECOND YEAR—FIRST TERM

1. The four fundamental operations: symbols of aggregation.
2. Factoring of the following types:

$$ax+bx-cx,\ m^2-n^2,\ x^2+px+g,\ ax^2+bx+c.$$

3. Fractions limited in the main to those having monomial denominators.
4. Simple equations, ratio and proportion.
5. Involution, square root. The formulas:
$A=\pi r^2$, $c^2=a^2+b^2$, $A=\sqrt{s(s-a)(s-b)(s-c)}$ area of a triangle, $4d^2=7h$ (distance seen from a height). $S=\tfrac{1}{2}gt^2$, $v=\sqrt{2gh}$.
6. Graphs of linear expressions, and their use as ready reckoners. Construction of linear equations when two pairs of values are given.

Graphs of linear equations. Positive and integral solutions of linear equations by means of graphs.

SECOND YEAR—SECOND TERM

1. Quadratic equations in one unknown. Problems.
2. Graphs of quadratic functions in one variable.

$$E.g.\quad \pi r^2,\ x^2-4x-5.$$

Graphical solution of quadratics of the type

$$x^2+px+q=o.$$

Description of graphs of expressions such as $20-3x-2x^2$. *E.g.* For what values of x does the expression *(i)* increase, *(ii)* decrease; for what values of x *(i)* is it always positive, *(ii)* always negative. What is the least value the expression can have?

3. Systems of equations in two unknowns, one equation being quadratic and the other linear. Problems.
4. Exponents and radicals. The meaning and use of fractional exponents. Reduction of radicals confined to transformations of the simplest type.

$$E.g.\quad \sqrt{162},\quad \sqrt{5/8},\quad \frac{a-\sqrt{b}}{a+\sqrt{b}}$$

DEMONSTRATIVE GEOMETRY

THIRD YEAR—FIRST TERM

Books I and II:

Euclid's dictum, "There is no royal road to geometry," still retains its pristine freshness.

DEMONSTRATIVE GEOMETRY

THIRD YEAR—SECOND TERM

Books III, IV, V:

If the geometry is not completed in a year it should be studied in a subsequent year until it is completed. Some theorists fancy that geometry can be done in less than a year. This is not in accord with the experience of the best teachers of the subject. The minimum essential requirement is the syllabus of the National Committee on Mathematical Requirements.

FOURTH YEAR

1. Review of algebra through quadratics.
2. Remainder theorem.
3. Factoring by grouping terms; factors of $a^3 \pm b^3$.
4. Construction of equations, the roots being given.
5. Systems of equations with three unknowns. Problems.
6. Simultaneous equations involving quadratics. Problems.
7. Proportion and variation.

8. Exponents and radicals. Radical equations.
9. Arithmetic and geometric series.
10. Logarithms. Application to compound interest.
11. Theory of quadratics, complex numbers.
12. Binomial theorem with application to compound interest. *E.g.* Find the amount of one dollar for 5 years, when deposited in a savings bank, interest 2% every six months.
13. Graphs of the type,

$$x^2+y^2=a^2,\quad x^2-y^2=a^2,\quad xy=a,\quad y^2=ax,\quad ax^2+by^2=c.$$

PRACTICAL TRIGONOMETRY

(Taken after Intuitive Geometry and one year of Algebra, or in fourth year.)

1. Definitions of sine, cosine and tangent for angles less than 180°, also the usual convenient forms of these definitions for either acute angle of a right triangle.
2. Solution of right triangles by natural and trigonometric functions from the data,
   (i) Two sides.
   (ii) Hypotenuse and a side.
   (iii) One side and either acute angle.
   (iv) Hypotenuse and either acute angle.
3. Law of Sines.
4. Area of a triangle in terms of two sides and their included angle.
5. Area of a triangle in terms of three sides.
6. Solution of triangles given one side and two angles. (Law of Sines.)
7. Areas of regular polygons given a side.
8. Heights and distances:
   (i) To find the height and distance of an inaccessible object on a horizontal plane by two angles of elevation taken at a measured distance from points in a straight line with the foot of the object.

(ii) To find the distance of an object on a horizontal plane from two observations made above the plane.

(iii) To find the height of an inaccessible object situated above a horizontal plane and its height above the plane.

(iv) To find the distance on a horizontal plane of an inaccessible object. (Law of Sines.)

CORRELATED MATHEMATICS FOR THE HIGH SCHOOL

By R. E. Baker, Commerce High School

About twenty-five years ago Professor Moore, of the University of Chicago, delivered his presidential address before the American Mathematical Society, the address which has so profoundly stimulated progressive thinking on the teaching of mathematics. It was he who first put forth the idea that high-school mathematics should be correlated. In the following school year a tentative program for correlated mathematics was formulated by the teachers of mathematics of the University and the University High School. Professor G. W. Myers was the chairman of the mathematics department and under his direction the curriculum of his department was radically changed. With the assistance of other members of the department, he published two textbooks dealing with mathematics in a general way. These books were "Myers' First-Year Mathematics" and "Myers' Second-Year Mathematics." Though these books were never widely used, much credit is due Professor Myers for this pioneer effort. In particular to him must be given the credit for first attempting a thorough breaking-up of the "water-tight-compartment" method of teaching arithmetic, algebra, geometry, and trigonometry.

The second stage of this experiment was the testing out of these courses in the classroom and the recasting of the material into greater uniformity. This part of the work was directed by Professor Ernest R. Breslich, who became head of the mathematics department of the University High School in 1912. The result of this work was published in the four volumes of correlated mathematics by Mr. Breslich that are so well known today and are so widely used in the high schools of the land. Since the publishing of these texts on general mathematics, others have taken up the task of rearranging the traditional courses into texts that embody an exposition of general or correlated mathematics. Among the many engaged in this work today are such notables as Schorling, Clark, Rugg, Cobb, Smith, Young, and Reeve. The last word has not yet been said in this field; it yet offers opportunity for much valuable work.

It is being argued today in Texas by some of the mathematics teachers and by others that our present curriculum in mathematics is out of date, is inadequate, does not measure up to the standards of progress that is being made in other departments, and that it should be discarded and in its stead be placed a curriculum of general mathematics. It is being recommended that algebra, geometry, arithmetic, trigonometry, and the easy part of analytics and the calculus all be combined into a general mathematics course and so taught in our Texas high schools. Many arguments are being given for a thorough reorganization, some of which point toward the major-errors of the current practices in secondary-mathematics instruction and others to the advantages offered by the correlated courses over the "compartment method."

The conventional algebra course is characterized by excessive formalism, and there is much drill work largely on non-essentials. In a correlated course this excessive formalism is greatly reduced. The emphasis is placed on those topics concerning which there is general agreement, namely, function, the equation, graph, and formula. The time thus gained permits more ample illustrations and applications of the principles and the introduction of more significant material, that is, material that is more closely related to the everyday life of the high-school pupil.

In correlated courses, instead of crowding the many difficulties of the traditional geometry course into one year, geometry instruction is spread over the years that precede the formal course, and the relations are taught inductively by experiment and measurement. This gives the pupil the vocabulary, the symbolism, and the fundamental ideas of geometry early in his school life. If he now leaves school or drops mathematics, he nevertheless has an effective organization of geometric relations.

The traditional courses delay the consideration of much interesting and valuable material that the field of secondary mathematics has to offer, and which may well be used to give the pupil very early an idea of what mathematics means and something of the wonderful scope of its application. The material of our seventh, eighth, and ninth years is often indefensibly meaningless when compared with that of many foreign curricula. Trigonometry, containing many easy and really practical problems, furnishes a good example of this delay. Other examples are found in the use of logarithms, the slide rule, standardized graphical methods, the notion of function, the common constructions of practical drawing, the motivation of precise measurement, a study of the importance of measurement in modern life, and the introductory ideas of the calculus. It appears that the mathematics student should be given an opportunity to use these important tools very early in his study. They lend to the subject a power and an interest that drills on formal material cannot possibly give.

Particular emphasis should be given early to graphical representation of statistics. The growing complexity of our social life makes it necessary that the intelligent reader possess elementary notions of statistical methods. The hundreds of articles in the current magazines so extensively read demand an elementary knowledge of these in order that the pupil may not be ignorant of the common everyday things of life. Brief chapters on the use of logarithms and the slide rule should be introduced early in the course in order that the pupil may be able to use these useful tools in his subsequent work, whether in the classroom or out in everyday life. Actual classroom experience with these labor-saving devices has proved them to be relatively simple and good material for eighth-grade and ninth-grade students.

The teaching of algebra, geometry, and trigonometry in separate fields is an artificial arrangement that does not permit the easy solution of problems concerning projects that correlate with problems met in the physical and biological sciences or the manual and fine arts. To reject the formalism of algebra, to delay the demands of a logical unit in geometry, and to present the simple principles of the various branches of mathematics in the introductory high-school course and next take up the more difficult principles and so on through the succeeding years of the high school gives the pupil a greater variety of problems that seem to be real applications. The pupil sees the usefulness of the various modes of treatment of the facts of quantity. Power is gained because the student is equipped with more tools, in that the method of attack is not limited to one field.

The aim of the general mathematics course should be to give the student *not so much mathematics* as is generally given in the traditional courses but *more about mathematics.* In other words, the student should know mathematics as a tool to be used rather than as a group of related subjects, such as algebra, plane geometry, solid geometry, trigonometry, and so on.

The question will be asked whether the results obtained with a course in correlated mathematics are as good as, or superior to, those attained by pupils trained in the traditional courses. From the report given by the teachers of the Chicago University High School in which the correlated course has been in use for several years, the following gains are made:

1. A saving of time. The work usually done in three and one-half years in separate courses is being done in three years, without noticeable loss to the pupil.
2. A large registration in mathematics. In this school 70–75% of all pupils are taking regularly courses in mathematics. In view of the fact that only one year is required for graduation this must be considered a high election.
3. Failures are reduced. The average number of failures in the course in mathematics for the last few years is less than 10%.

4. Successful college work. The University High School students, who have completed four years of mathematics, take calculus as their first course in the University and are able to carry the work successfully. Reports from other colleges are also very favorable, showing an average grade of *B* in all mathematics courses taken by pupils trained in high school by correlated methods.

5. The teaching of material relatively simple in the introductory courses, which in the traditional procedure are delayed so as to be taught in the later years of the high school when but few students are afforded an opportunity to master them.

6. Greater power in problem solving through the use of the notion that geometry constitutes concrete material in which the situation is graphic and relatively easy.

Other schools that have tried this correlated course give the report that they have met with none other than the greatest success. The teachers are all very much pleased with the work, and the pupils seem to enjoy the courses to a much greater extent than they did the conventional divorced courses. The number continuing to elect mathematics through the entire four years of high-school life has increased, as reported by several schools, by more than 20% since the change to correlated mathematics has been made. After a thoughtful consideration of these statements, the writer believes that a mathematics course as described would function to an advantage in our Texas high schools.

"ALGEBRAIC CUT-OUTS"

By Miss Rebecca Goldstein, El Paso

If Johnny sits with his mouth open, we whisk him off to a surgeon and have his adenoids out; if he has growing pains, we throw a searchlight on his teeth, and woe betide the hapless germ that is found lurking there. Out come his teeth and with them, theoretically, all the ills that man is heir to. If his throat is sore instead of using a piece of red flannel with a generous slab of bacon or a dash of kerosene to wrap around his neck, we treat him instead to a "tonsilotonomy" and the fact is recorded in the society column. If, after running all day, he complains of a pain in his side, we cry "Out with his appendix! Fie upon it!" and lo! he becomes once more a hero, having boasted this time a *bona fide* major operation.

We live in an age of cut-outs—perhaps the automobiles are responsible for the term (at least they've made it audible)—or was it the movie censor, that surgeon of the cinema, so to speak? Since with Spartan fortitude we diet and cut out certain articles from the food for our bodies, what could be more natural, I ask you, than that we educational dietitians get busy and cut out undesirables from the list of mental foods? Surely there'll be no bewailing there. Johnny may object to having sweets eliminated from his diet, but when partial payments and compound interest are cut out of his arithmetic, Johnny does a Charleston. We can be sure that education is doing at least one thing that Johnny approves of. Let us catch some of his enthusiasm and if we're too old (?) or too dignified to execute a Charleston, let us at least rejoice, for this is one feature of modern pedagogy that is a boon to each end of Mark Hopkin's famous log—algebra's well-known process of elimination is to react as a boomerang upon itself and the youth of this generation is enjoying "this freedom" almost to infinity—or is that term taboo also?

Seriously speaking, why cut out any of our traditional subject matter? It's our job as mathematics teachers to give the pupils of our high school the type of algebra that is vitalizing and useful, though not necessarily always immediately practical. Even under pressure of an over-crowded curriculum, it's up to us to make possible the achievement of satisfactory results in modified traditional subject matter.

We need to make mathematics simple; make mathematics clear; make mathematics appear easy although it may be difficult. Given more time and less to do, surely we can give more permanent knowledge and skill to the pupils in the topics taught.

The question, though, is "What should we cut out?" This has been discussed many times, but has a definite settlement ever been reached? Not yet. We all agree, do we not? that some of our traditional mathematics should be wrecked—topics which are obsolete, valueless and not necessary for the further study of applied mathematics. In the wrecking process, let us begin with "nests of parentheses"—say above two—do you not agree? Then H.C.F. and L.C.M.—except incidentally with addition of fractions and in solving fractional equations. With the pupil's knowledge of this topic from the corresponding processes in arithmetic any L.C.M. ever needed can be found by inspection.

Rarely ever is it necessary to factor expressions of as many as six terms, is it? Not often enough to prevent our putting a taboo on all those over five terms. Then the "awful" complicated complex fractions, those more than "four stories" in height, any with unusual denominators, cut these out. Johnny will heave a sigh of relief and never again will mathematics be drudgery.

Also, what difference does it make at what time between three and four o'clock the hands of a clock are at right angles to each other? Why bother about how many months it will take to fill a cistern or any other artificial problems as "digging and digit problems" as long as we can find many real problems within the experience of the pupil and not have Algebra a "dry as bone" subject?

If determinate equations could be thrown overboard, variation might be included in ratio and proportion merely as an application. Then infinite series, I'd say "into the excess baggage" class; and to avoid pitfalls I'd dabble with division by zero but leave that for the college to teach.

'Twould be an ill wind indeed that would not unquestionably blow some good to the students were these things eliminated from our high-school course. Johnny could then be given graphs—statistical graphs—bar, circle, line; formulas, and more formulas. Applications of formulas to science and industry, formulas found in books on radio, approximate computations, verbal problems related to things in which students are interested and with which they are familiar—problems relating to I. Q's, football teams, airplanes—live problems within their everyday experience. Let's make algebra real and interesting to the student right from the outset. Let's make the course so attractive that our students will actually *love* algebra. But first we must love it ourselves. Let's not get discouraged when our students come to us seeming to know nothing about anything. Remember:

It's easy enough to be pleasant
When pupils come to us well taught,
But the teacher worth while
Is the one who can smile
When the sum of their knowledge is naught.

DENVER PUBLIC SCHOOLS

For several years committees of teachers and supervisors have been working in the Denver city schools under the direction of specialists revising the course of study and making outlines in each grade and in each field. The result of the work is most excellent. The course of study for the Junior High School has been advanced to the stage that it is now in print, but only a tentative outline for the Senior High School has yet been made. The active coöperation of the great majority of teachers and administrative officers in such a thoroughgoing piece of work must of necessity be a great stimulus in the development of good teaching and sound scholarship. Every progressive Junior High School teacher should be acquainted with what is being done in Denver. It would be well for the teaching corps of every public school system to have committees working in a similar way on curriculum and content of courses. It gives a broader outlook to the individual teacher and overcomes the deadening influence of the narrowness that comes to the grade and high-school teacher whose work is confined to the four walls of his classroom. We regret that we have not space to give the complete report on high-school work, but in what follows there will be noticed some of the best features:

JUNIOR HIGH SCHOOL

Grade 7 B.—This half-year is full and rich in outline of arithmetic and intuitional and experimental geometry. The arithmetic is a continuation and application of what has preceded with a thorough treatment of percentage.

Grade 7 A.—In this grade the arithmetic is continued but formulas are introduced for percentage and interest. Letters represent numbers and now become useful in the statement of rules and principles. Perimeters and areas are given by the expressions: $P=a+b+c$; $A=bh$; $A=\pi r^2$; $C=\pi d$. These are followed by numerous exercises and suggestions. Similar work is given with volumes of prisms, cylinders, cones, spheres, etc.

Grade 8 B.—Review of arithmetic conducted frequently during the year. Algebraic language, using formulas whose meaning is understood. Show advantage of symbols of algebra in the expression of rules. Plus and minus sign to have meaning as in arithmetic. Review areas, volumes, etc., using letters. Show use of equation in stating and solving simple problems. Solve equations using four laws. Solution of many problems. Simple use of similar triangles leading to elementary trigonometry, finding height of trees, towers, kites, etc.

Grade 8 A.—Review of percentage, interest, and business problems. Positive and negative numbers. It is to be noticed especially that several months' training in the use of letters before the negative number is given. Negative and positive numbers illustrated by playing games, debts and money earned, latitude and longitude, forces in opposite directions, measuring temperatures, etc. Addition, subtraction, multiplication, division of positive and negative numbers: first by use of four numbers and then with letters.

Grade 9 B.—Relations of quantities, leading to graphical methods. Equations and use of axioms. Statement of problems and solution of equations—checks scientific laws stated by formulas. The right triangle and trigonometry ratios. Measurement of heights and distances across streams. Angles and triangles, ratios, trigonometric ratios. Fundamental operations with negative numbers. Use of terms, factors, parentheses factoring.

Grade 9 A.—Algebraic fractions. Fractional equations, literal equations. Use and construction of formulas. Graphical methods, coördinate axes, points of intersection. Inconsistent equations. Use of axioms in elimination. Elimination by substitution, check solutions. Variation and proportion. Roots and powers, square root, cube root. Powers and exponents, meaning of fractional negative and zero exponents. Logarithms and exponents, slide rule quadratic equations and their solutions. Graphs of the quadratic, statistics.

The above is a bare outline of the many suggestions and valuable material mentioned in the course of study. It will show to the reader, the most excellent way in which the early notions of algebra should be approached.

SENIOR HIGH SCHOOL

The tentative outline of the course of study for the senior high schools of Denver is prefaced by the following interesting observations:

> In the last twenty-five years the disciplinary value of mathematics has been much discussed. Once the disciplinary value of arithmetic, algebra, and geometry was unquestioned and these subjects were required by all high schools and colleges, then the stronghold of these time-honored subjects was shaken by the belief of some psychologsits that the value gained from one subject is not carried over into any other subject; but now a more sane and rational theory than either of these extremes prevails, namely, that there is a disciplinary value in mathematics for all those who understand the subject and derive pleasure in its study, and the number of these is by no means small.
>
> The best workers in mathematics believe that in addition to the great practical value of the subject is the transfer of analytical reasoning to science, law, medicine, business and government.
>
> While the disciplinary value of mathematics is recognized by psychologists as existing, and is conceded by them to be a valid reason for teaching mathematics in the Senior High School, its disciplinary value depends greatly upon the method of teaching. The teacher of mathematics must meet the challenge of the psychologist to give the kind of training in logic and right thinking that will carry over into other subjects and into life.
>
> A man is cultured if he has the general knowledge of human affairs that comes from reading, travel, and social intercourse; if he knows, broadly speaking, the steps through which the race has reached its present civilization; if he knows enough of science to read intelligently the references to biology, chemistry, and physics that occur in our papers and magazines; if he has an appreciation of art as expressed in painting, architecture, music, and drama. Mathematics has played so large a part in the advancement of mankind from the counting of herds by pebbles (Calculus) to the invention of the decimal system with zero to stand for the absence of a number and to computing by logarithms; from the construction of the pyramids of Egypt and the temples of the Nile and of Greece to the Gothic Cathedrals whose very ornaments are geometric designs, and to the skyscrapers of

New York. All mechanical means of transportation are applied mathematics; all finance, public and private, is mathematical. Surely a man is not cultured in the broadest sense if he has no knowledge of the subject upon which our present civilization of great mechanical development is based.

And just as a knowledge of history and literature is essential to culture because they are important parts of human knowledge and because they give us a better insight into social conditions, just so mathematics is also necessary to culture. From this viewpoint mathematics is essential, because it is a piece of truth upon which the development of civilization has at all time been dependent.

There is not today a single line of commercial work in which the formula is not, or may not be, profitably used, and without a knowledge of the formula, of its value, and of its manipulation by the aid of the equation, a man cannot be a real master of any great commercial field.

This knowledge of the formula is, however, no more vital than a knowledge of other parts of algebra, as for example, the graph in its varied forms. Here is one of the several tools of mathematics which every business man needs and actually uses in his everyday business life. Algebra, then, may be said to be good training for a commercial life.

The plotting of statistics covering long periods of years is coming more and more to be regarded as the scientific method of forecasting commercial events.

The absolute necessity of the intensive study of algebra, geometry, and trigonometry by pupils, who are going into engineering and mathematical sciences, is unquestioned. Mr. John R. Clark says that pupils who are in technical pursuits are constantly being confronted with the fact that there is an equation for every law. Algebra is the science of the equation. The pupil who elects mathematics and who does not go to college or does not enter a profession largely based on mathematics must get out of the course a disciplinary and cultural value that is useful to him.

But the primary objects in the study of geometry have not been attained until the pupils have received training in clear and logical thinking and have become reasonably proficient in expressing their thoughts correctly and to the point.

The material for discussion in social science may be of the greater practical value to the future citizen but it is not so well fitted for training his immature mind in logical reasoning.

Geometry offers the best material for training the immature mind to reason logically; for the facts involved are few and simple; all irrelevant matter is easily eliminated; nothing is a matter of opinion, true today and false tomorrow, right for one people and wrong for another, but that which is true for geometry once is always true.

In order to accomplish this training in logical reasoning, attention must be given both to analysis and synthesis. The set demonstrations are patterns of the latter and as such are valuable. The original exercises, on the other hand, afford the best training. Their solution depends upon a careful analysis of the conditions, a sifting out of the irrelevant, a gathering together of related facts, an application of known truths to new situations, and finally, a building up of the argument into a synthetic proof. Analysis can proceed with more ease in geometry than in any other school subject because the magnitudes involved are within the pupil's experience. Greater attention should be given to original exercises than to the proofs of the set propositions.

Grade 10.—The year is given to the study of plane geometry with a possibility of certain phases of solid geometry. A list of necessary theorems has been prepared with a view to revision after a thorough trial has been made. In this way, the year in geometry is to be finally worked out. The work in demonstrative geometry is to be accompanied with exercises and examples. In a course of study that has had, up to this point, all subjects so well correlated, one is led to wish for more applications of algebra while this year of training in demonstrative geometry is given. This is probably the most difficult problem to be met in the high-school curriculum.

Grade 11 B.—Intermediate algebra. The work of this year covers the subjects noticed in *9 A*, but in a more advanced and mature way. The outline of the work of this grade is full of interest. It shows how the elementary principles of the first course in algebra may be repeated and at the same time give a broader and more thorough treatment. A part of the time of the grade is devoted to trigonometry.

Grade 11 A.—The work begins with the quadratic equation. Solutions by completing the square, by formula and by factoring. Graphs of linear, quadratic and cubic equations. Solution of quadratic equations by graphs, character of the roots. Form of quadratics, simultaneous quadratics, graphical solutions. Coördinates. Exponents and radicals, logarithms, powers and roots. Exponential equations, slide rule, complex numbers. Arithmetic and geometric progression. Business problems. Study the graphics of the circle, ellipse, parabola, hyperbola.

Grade 12 B.—Solid geometry. If possible some of this work is to be given in the tenth grade, giving time for analytics or other more advanced topics.

Grade 12 A.—Trigonometry formulas, identities, solutions, use of logarithms.

TEXTBOOKS IN HIGH-SCHOOL MATHEMATICS

By J. E. Burnam, Simmons University, Abilene

The next adoption of textbooks in algebra and geometry for use of Texas schools will be made in October, 1927. Thus, before the next meeting of our State Teachers' Association, our books will have been selected for another period of years, and it behooves us to bestir ourselves if we are concerned about the tools with which we must attempt to teach our subject. The Textbook Commission will, of course, be showered with tentative texts, good, bad, and indifferent, each bearing out some alleged revolutionary idea in the pedagogy of mathematics, or some special hobby of its author, or written merely for the hope of the emoluments that accrue to the author of the successful candidate for adoption.

The teachers of the State must make it their business to see that the text adopted is in accord with their ideas and desires in so far as it is possible for one text to meet the needs of such varied interests as are to be served, interests extending as they do from the remote rural schools, with their peculiar environment and aims, to the villages and cities with their different needs and viewpoints. It is, of course, evident that no one text could conform in detail to these varied needs nor could one text conform to the ideas of the widely varying group of teachers who are to use it. But we can all agree as to the general features of what will constitute a good text, and the only purpose of this paper is to attempt to point out some of those features, without any brief for or against any particular text or author. The recommendations made are in accord with the conclusions of the leading investigators and writers on the pedagogy of mathematics, including Charters, Inglis, Judd, Hines, Breslich, and the Bulletin of the Mathematical Association of America entitled "Reorganization of Mathematics in Secondary Education."

The use and place of mathematics in our educational system and the ends to be sought in teaching this subject, will, or should determine the sort of texts to be chosen. The uses of mathematics in general are usually set forth under three headings: (a) practical, (b) disciplinary, (c) cultural. To quote from the bulletin of the national committee: "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and space which are necessary to an insight in and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual."

On the side of algebra, the ability to understand its language and use it intelligently, the ability to analyze a problem, to formulate it, and to interpret the result must be dominant aims. Drill in algebraic manipulations should be limited to those processes required for a thorough understanding of the principles and for probable applications to be met later, either in common life or in subsequent courses which a large proportion of the students will take. Through the seventh, eighth and ninth grades, it is the recommendation of the national committee that the course for these three years be planned as a unit with the purpose of giving the pupil the most valuable training he is capable of receiving during those years, with little reference to courses which he may or may not take in later years. There appears to be no conflict of interest during this period between those who ultimately go to college and those who do not, and college entrance requirements should receive no specific consideration.

As to the question of making algebra practical, it is doubtful if the topics in the traditional algebra course will find application in applied problems of a materially different type or of much wider variety than we now have. The greatest promise in the direction of a good supply of real problems may come in finding new types of application. For example, graphic representation and statistics are new sources of useful material. The algebra of thrift, investment, and insurance offer possibilities.

Throughout an up-to-date algebra course, there will be a constant development and exercise of the function concept, including the important notions of a mathematical law and the great service algebra has performed in the discovery and application of such laws. There will be throughout the course more emphasis on straight thinking, with less on mere mechanical drill. It is not too much to hope that algebra, if properly taught, will have genuine disciplinary values, in developing patience to go through a mass of details, keeping each item in mind, the ability to do close and detailed thinking, to give sustained attention, to analyze and to draw conclusions.

In content the algebra course should adhere reasonably closely to the recommended course of the bulletin referred to. It should give heed to the modern tendency toward unified mathematics as sponsored by Breslich and others.

The unit idea as advanced by Professor H. C. Morrison, of the University of Chicago, should apply in a general way to our new algebra text. This idea is comparable to the so-called project method and topic method being used successfully by teachers of literature, history, geography, etc. A unit must be a comprehensive and significant aspect of the subject. It must be teachable as well as learnable and must be capable of being tested for mastery. It must be comprehensive. For example, addition and subtraction of denominate numbers in arithmetic may be most economically taught in connection with decimals. Addition, subtraction and the ability to discriminate between the two form a single unit. Multiplication and division constitute a unit but even better grasp comes by adding the notion of variation which will clear up the question too often asked, "Do you multiply or divide?" Each unit requires time and patience and skillful technique but time required is of less importance than the assurance of learning. Do away with the traditional "So many weeks of this, so many weeks of that."

Arithmetic of the first six grades falls into about nineteen or twenty units, according to Professor Morrison's classification. Algebra, geometry, and trigonometry are subject to the same treatment. In fact, the idea of unified mathematics as advanced by Breslich and others is in conformity with the unit idea. In general education mathematics is valuable chiefly as a means of interpreting those aspects of the world which are not otherwise capable of analysis. In each unit of mathematics, therefore, the material may be said to be made up of applications in the form of what are commonly called problems, each focused upon the unit, and coming within the pale of children's experience as completely as possible. There need be very few of the mere puzzles, clock problems, hare and hound, fast and slow train, tank, and others so familiar to the traditional text. Rather there will be problems which appeal to the intuitive sense of geometrical and trignometrical relations, problems of measurement, the interpretation of measurable relations reduced to formulae and the like. Problems on the speedometer and calculating machine would be better than the traditional digit problems.

Applying the unit notion to algebra Professor Morrison indicates the following twenty-two units as suitable organization for the algebra course:

> 1. Concept of literal numbers. 2. Plus and minus quantity. 3. Graphic representation. 4. The equation of simple linear type. 5. Addition and subtraction. 6. Multiplication. 7. Expansion of binomial and trinomial expressions. 8. Division. 9. The concept of factors. 10. The factoring process as applied to the class most commonly used. 11. The algebraic fraction as a ratio. 12. Lowest terms. 13. Fractions greater than unity and mixed numbers. 14. The common denominator. 15. Addition and subtraction of fractions. 16. Multiplication and division of fractions. 17. Linear equations with one unknown, with proportion as an added item of assimilated material. 18. The linear equation with two or

more unknown. 19. Functions, linear and quadratic graphing. 20. The quadratic equation. 21. Formulas as algebraic representations. 22. Square root.

If the student has completed a course in algebra as thus outlined, he will come to the study of formal geometry with many of the notions of space relation already established in his mind through their use in the problems of the texts in arithmetic and algebra. The geometry course should conform to the recommendations in the bulletin above referred to. Geometry and trigonometry lend themselves to the unit treatment. Professor Morrison suggests the following roganization of the geometry course:

1. The angle: concept of angle, measurement, right angle, sum about a point, complementary and supplementary angles. 2. Parallel lines. 3. Perpendicular lines. 4. The triangle as a plane figure, sum of angles, types of triangles, exterior angle relations, properties of right, isosceles, and equilateral triangles, congruence, similarity, relations of angles and sides. 5. Quadrilaterals. 6. Polygon. 7. The circle as a plane figure: angle values, angles and arcs, tangency, chords and arcs, inscribed angles, angle relations of chords, tangents and secants, relation of circle to inscribed and circumscribed polygon. 8. Areas. 9. Loci. 10. Proportion as applied to plane figures. 11. Symmetry.

In conclusion, it is here recommended that the texts adopted be in conformity to the sound results of modern research. Let the teachers of the State inform themselves as to these findings and then insist with every ounce of pressure we can bring to bear that these findings be incorporated in our textbooks.

METHODS OF DEFINING THE TRIGONOMETRIC FUNCTIONS

MARY E. DECHERD

A few remarks about the definitions of the trigonometric functions may not be out of place in our *Bulletin*. Dr. L. L. Smail, in the preface to his "Plane Trigonometry" says: "Textbooks and teachers are about evenly divided on the question of defining the trigonometric functions, some preferring to define them for acute angles first and for the general angle much later, others insisting on the general definition at the outset." While I do not question the truth of Dr. Smail's statement, my own experience would lead me to believe that there are many more of both teachers and textbooks in the former class than in the latter.

For many years, I followed the textbooks which I happened to be using; and as every one of these texts defined the functions for acute angles first, I began the course in trigonometry by defining the functions for acute angles. I have now come to the conclusion that, while it may not be a matter of primary importance in the teaching of trigonometry, nevertheless there is much to be gained by employing the other method of procedure, and defining at the outset of the course the functions of the general angle.

In the first place, there is, I think, a saving of time if the functions of angles in all four quadrants are defined at once. The student beginning trigonometry has already become familiar in algebra with the x and y axes and with the coördinates of points in each of the four quadrants, and hence is acquainted with the terms needed in giving these definitions. Draw the four following figures:

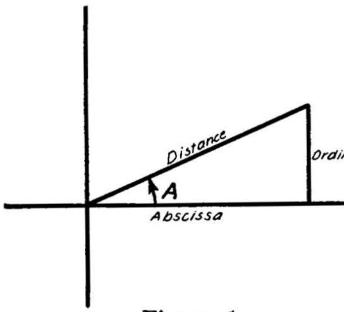


Figure 1

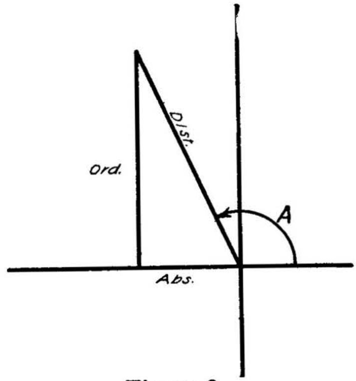


Figure 2

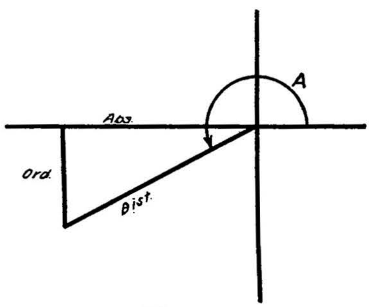


Figure 3

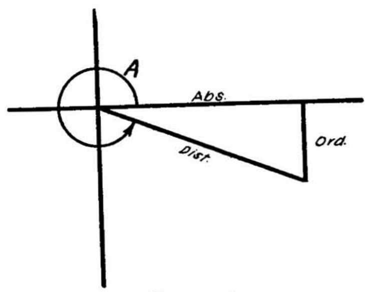


Figure 4

In each of the four quadrants

sine $A = \dfrac{\text{distance}}{\text{ordinate}}$ cotangent $A = \dfrac{\text{abscissa}}{\text{ordinate}}$

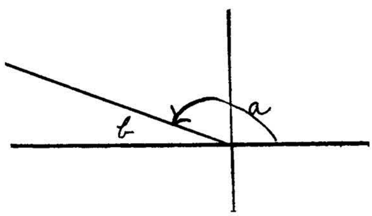
cosine $A = \dfrac{\text{abscissa}}{\text{distance}}$ secand $A = \dfrac{\text{distance}}{\text{abscissa}}$

tangent $A = \dfrac{\text{ordinate}}{\text{abscissa}}$ cosecant $A = \dfrac{\text{distance}}{\text{ordinate}}$

From these definitions the functions can be derived for 30°, 45°, 60°, 120°, 130°, 150°, 210°, 225°, 240°, 300°, 315°, 330°. By the time the student has derived the functions of these angles (by drawing the figures), or in some other way applied the definitions of the functions, he has learned how to define the functions of *any* angle.

In the second place, it frequently results if the functions of acute angles are given first, that the student never lays aside the terms "adjacent side," "opposite side," and "hypotenuse." He forgets that there are no "opposite" and "adjacent" sides for the angles in the second, third and fourth quadrants. If he is asked to give the tan of an angle in Quad 2, he invariably says it is $\frac{\text{opposite}}{\text{adjacent}}$ not realizing that his terminology is meaningless. It is, I think, bad policy to use a set of terms in defining functions in Quad 1 that cannot be employed correctly in the other quadrants.

In the third place, one of the most common and most insidious errors of students in trigonometry is to think that the angle referred to in the second, third, and fourth quadrants is the acute angle in these quadrants. I do not greatly wonder that the student thinks he talking about angle *b*, when he is allowed to say that tan $a = \frac{\text{opposite side.}}{\text{adjacent side.}}$. The fact that the functions of the acute angle are defined first also tends to develop in the students the idea that trigonometry is primarily concerned with acute angles and hence encourages him in thinking we are talking about *b* when really we are discussing *a*. If we must emphasize the acute angle by giving the definitions in terms of the sides of the right triangle, do this after the functions of the general angle are defined and not previously.



Hence, because it will really save time, because the reverse procedure leads to misconceptions, and because it is no more difficult, I would record myself as being in the group of "teachers insisting on the general definitions at the outset."